

Phase ambiguity of the threshold amplitude in $pp \rightarrow pp\pi^0$

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Abstract

Measurements of spin observables in $pp \rightarrow \vec{p}\vec{p}\pi^0$ are suggested to remove the phase ambiguity of the threshold amplitude. The suggested measurements complement the IUCF data on $\vec{p}\vec{p} \rightarrow pp\pi^0$ to completely determine all the twelve partial wave amplitudes, taken into consideration by Mayer et.al. [15] and Deepak, Haidenbauer and Hanhart [20].

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I. INTRODUCTION

Meson production in NN collisions has continued to excite considerable interest [1–4], since total cross-section measurements [5] for $pp \rightarrow pp\pi^0$ in the early 1990's were found to be more than a factor of five larger than the then available theoretical predictions [6]. To bridge the gap between experiment and theory, several mechanisms like, exchange of heavy mesons, two pion exchange, off shell extrapolation of the vertex form factor, final state interactions, contributions due to Δ resonance and of low lying nucleon resonances, were proposed. Hanhart et al. [7] in 2000 have observed: “As far as microscopic calculations of the reaction $NN \rightarrow NN\pi$ are concerned, one has to concede that theory is definitely lagging behind the experimental sector....Further more they take into account only the lowest partial wave(s). Therefore, it is not possible to confront these models with the wealth of experimental data available nowadays with differential cross-sections and with spin dependent observables.” The Julich model, on the other hand, takes into consideration higher partial waves as well.

In contrast to elastic NN scattering where channel spin is conserved, the $pp \rightarrow pp\pi^0$ transition at threshold to the final Ss state is a triplet to singlet. Next in order are the transitions to Ps states which are singlet to triplet. As the energy is increased transitions to Pp states are also expected to contribute, which are, however, triplet to triplet. Pionic d -wave effects were reported [8] even at a beam energy of 310 MeV. Measurements upto 425 MeV have also been reported [9], where evidence for Ds state was seen even at 310 MeV. Advances in storage ring technology [10] led to detailed experimental studies, including measurements of spin observables employing polarized beams of protons on polarized proton targets. Of the two existing models [11, 12] which include higher partial waves, the Julich meson exchange model [7, 11] was thoroughly confronted with these data. The model was comparatively more successful with the less complete data on $\vec{p}\vec{p} \rightarrow d\pi^+$ [13] and $\vec{p}\vec{p} \rightarrow pn\pi^+$ [14], but failed to provide an overall satisfactory reproduction of the complete set of polarization

observables in the case of $\vec{p}\vec{p} \rightarrow pp\pi^0$ [15]. In this context, a model independent approach [16, 17], was developed, using irreducible tensor techniques [18]. The reaction is characterized, in this formalism, by irreducible tensor amplitudes $M_\mu^\lambda(s_f, s_i)$ of rank $\lambda = |s_f - s_i|, \dots, (s_f + s_i)$, where s_i, s_f denote the initial and final spin states of the two protons. Each of these amplitudes is expressible in terms of partial wave amplitudes $M_{l(l_f s_f)j_f; l_i s_i}^j$, which are functions of the c.m. energy E and invariant mass W of the two proton system in the final state. The relative orbital angular momentum between the two protons in the initial and final states are denoted by l_i and l_f respectively and l denotes the pion orbital angular momentum in the c.m frame. The threshold amplitude $M_{0000;11}^0$ contributes to $M_0^1(0,1)$, and an empirical estimate of the integrated $|M_0^1(0,1)|^2$ was presented in [16], based on the then existing data [5]. The same approach was employed subsequently to analyze [19] the IUCF data on $\vec{p}\vec{p} \rightarrow pp\pi^0$ [15] immediately after its publication. The sixteen partial waves listed by Mayer et al. [15], covered the Ss, Ps, Pp, Sd and Ds channels. Here, the capital letters denote l_f while the lower case indicate l_i . In [20], the same set of partial waves were listed, of which, the last four, covering Sd and Ds were ignored following [15]. Since, the final spin-singlet and spin-triplet states do not mix in any of the spin observables measured in [15], the Ss amplitude and the larger of the Ps amplitude were both chosen to be real in [20]. This implies that the phase of the Ss amplitude remained ambiguous but chosen to be zero with respect to the larger Ps amplitude. The comparison of the empirically extracted amplitudes with the Julich model predictions revealed that *i*) the Δ contributions are important, *ii*) the model deviated very strongly in the case of ${}^3P_1 \rightarrow {}^3P_0p$ and to a lesser extent in ${}^3F_3 \rightarrow {}^3P_2p$ which “will guide the search for the possible shortcomings.” [20]

The purpose of the present paper is to extend the model independent theoretical discussion to the spin polarization of the protons in the final state and to examine how the additional experimental measurements regarding the final spin state can be used to determine empirically, the strengths of all these amplitudes and the ambiguous

relative phase of the threshold Ss amplitude with respect to the eleven near threshold Ps and Pp amplitudes, considered in [15, 19, 20]

II. THEORETICAL FORMALISM

We consider the reaction $pp \rightarrow pp\pi^0$ at c.m. energy E and initial c.m. momentum $\mathbf{p}_i = p_i \hat{\mathbf{p}}_i$ which may be chosen to be along the z -axis. Let $\mathbf{q} = q\hat{\mathbf{q}} = -(\mathbf{p}_1 + \mathbf{p}_2)$ denote the pion momentum in c.m. frame and let $\mathbf{p}_f = p_f \hat{\mathbf{p}}_f = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2)$ in terms of the c.m. momenta \mathbf{p}_1 and \mathbf{p}_2 of the two protons in the final state.

Following [16], we write the matrix M in spin space for the reaction $pp \rightarrow pp\pi^0$ in the form

$$M = \sum_{s_i, s_f=0}^1 \sum_{\lambda=|s_i-s_f|}^{s_i+s_f} (S^\lambda(s_f, s_i) \cdot M^\lambda(s_f, s_i)), \quad (1)$$

where s_i and s_f denote the initial and final channel spins respectively. The irreducible tensor operators $S_\mu^\lambda(s_f, s_i)$ of rank λ with μ taking values $\mu = \lambda, \lambda-1, \dots, -\lambda$ are defined in [18]. The irreducible tensor amplitudes $M_\mu^\lambda(s_f, s_i)$ in (1) are expressible as

$$M_\mu^\lambda(s_f, s_i) = \sum_{\mathcal{L}, j} W(l_i s_i L_f s_f; j \lambda) Z(s_f, s_i, \mathcal{L}, j) \times A_\mu^\lambda(\mathcal{L}), \quad (2)$$

where

$$A_\mu^\lambda(\mathcal{L}) = ((Y_{l_f}(\hat{\mathbf{p}}_f) \otimes Y_l(\hat{\mathbf{q}}))^{L_f} \otimes Y_{l_i}(\hat{\mathbf{p}}_i))_\mu^\lambda, \quad (3)$$

and the symbol \mathcal{L} is used to collectively denote $\mathcal{L} \equiv \{l_f, l, L_f, l_i\}$. It may be noted that $(-1)^{l_f+l+l_i} = -1$ due to parity conservation. The complex numbers $Z(s_f, s_i, \mathcal{L}, j)$ are given by

$$Z(s_f, s_i, \mathcal{L}, j) = \frac{[L_f][j]^2}{[s_f]} (-1)^{j-s_i+1} \sum_{j_f} [j_f] \times W(s_f l_f j l; j_f L_f) M_{l(l_f s_f) j_f; l_i s_i}^j \quad (4)$$

in terms of the sixteen partial wave reaction amplitudes

$$M_{l(l_f s_f) j_f; l_i s_i}^j = F \langle (l(l_f s_f) j_f) j | T | (l_i s_i) j \rangle, \quad (5)$$

proportional to the reduced on-energy-shell T matrix elements $\langle (l(l_f s_f) j_f) j | T | (l_i s_i) j \rangle$ for the reaction. The purely kinematical factor

$$F = (-i)^{l_i-l-l_f} 4(2\pi)^{1/2} \sqrt{W \omega (E - \omega) q p_f / p_i}, \quad (6)$$

is introduced explicitly in (5), so that the dependence on E and W is seen to be completely taken care of by the $M_{l(l_f s_f) j_f; l_i s_i}^j$. They are identical to the amplitudes denoted as T in [20]. We may, following [15, 20], neglect the last four amplitudes which are Sd and Ds and consider

TABLE I: List of the partial wave amplitudes for the reaction $\bar{p}\bar{p} \rightarrow pp\pi^0$

Initial pp state	Type	Final $pp\pi^0$ state	Partial Wave Amplitudes
3P_0	Ss	$^1S_{0,s}$	$M_{0(00)0;11}^0 = f_1$
1S_0	Ps	$^3P_{0,s}$	$M_{0(11)0;00}^0 = f_2$
1D_2		$^3P_{2,s}$	$M_{0(11)2;20}^0 = f_3$
3P_0	Pp	$^3P_{1,p}$	$M_{1(11)1;11}^0 = f_4$
3P_2		$^3P_{1,p}$	$M_{1(11)1;11}^2 = f_5$
3P_2		$^3P_{2,p}$	$M_{1(11)2;11}^2 = f_6$
3F_2		$^3P_{1,p}$	$M_{1(11)1;31}^2 = f_7$
3F_2		$^3P_{2,p}$	$M_{1(11)2;31}^2 = f_8$
3P_1		$^3P_{0,p}$	$M_{1(11)0;11}^1 = f_9$
3P_1		$^3P_{1,p}$	$M_{1(11)1;11}^1 = f_{10}$
3P_1		$^3P_{2,p}$	$M_{1(11)2;11}^1 = f_{11}$
3F_3		$^3P_{2,p}$	$M_{1(11)2;31}^3 = f_{12}$

the first twelve amplitudes, which are, for simplicity, enumerated as f_1, \dots, f_{12} in Table I.

The unpolarized double differential cross section may now be written as

$$\frac{d^2\sigma_0}{dW d\Omega_f d\Omega} = \frac{1}{4} \text{Tr}[M M^\dagger], \quad (7)$$

where M^\dagger denotes the hermitian conjugate of M given by 1). The invariant mass W of the two protons in the final state is given by

$$W = \sqrt{(E^2 + m_\pi^2 - 2E\omega)}, \quad (8)$$

where m_π denotes the pion mass and ω denotes the c.m. energy of pion. It may be noted that

$$\frac{d^2\sigma_0}{d^3p_f d\Omega} = \frac{W}{p_f} \frac{d^2\sigma_0}{dW d\Omega_f d\Omega}, \quad (9)$$

It is worth noting that the threshold Ss amplitude f_1 alone contributes to

$$M_\mu^1(0, 1) = \frac{1}{4\sqrt{3}\pi} f_1 Y_{1\mu}(\hat{\mathbf{p}}_i), \quad (10)$$

which is spherically symmetric both w.r.t $\hat{\mathbf{p}}_f$ as well as $\hat{\mathbf{q}}$ in the final state, while all the other irreducible tensor amplitudes are independent of f_1 .

III. FINAL STATE POLARIZATION WITH INITIALLY UNPOLARIZED PROTONS.

If the colliding protons are unpolarized, the spin density matrix ρ^f characterizing the two protons in the final state is given by

$$\rho^f = \frac{1}{4} M M^\dagger, \quad (11)$$

so that (7) is identical with $Tr[\rho^f]$.

The final spin state is completely determined through measurements of the polarizations

$$\mathbf{P}_i = \frac{Tr[\boldsymbol{\sigma}_i \rho^f]}{Tr[\rho^f]}, i = 1, 2 \quad (12)$$

of the two protons and their spin-correlations

$$C_{\alpha\beta} = \frac{Tr[\boldsymbol{\sigma}_{1\alpha} \boldsymbol{\sigma}_{2\beta} \rho^f]}{Tr[\rho^f]}, \alpha, \beta = x, y, z. \quad (13)$$

All these spin observables may elegantly be calculated by considering

$$P_\mu^k(s'_f, s_f) = Tr[S_\mu^k(s'_f, s_f) \rho^f], \quad (14)$$

where $S_\mu^k(s'_f, s_f)$ are given in terms of the Pauli spin matrices $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ of the two protons in the final state through

$$S_0^0(0, 0) = \frac{1}{4}(1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \quad (15)$$

$$S_0^0(1, 1) = \frac{1}{4}(3 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \quad (16)$$

$$S_\mu^1(1, 1) = \frac{\sqrt{3}}{2\sqrt{2}}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)_\mu^1 \quad (17)$$

$$S_\mu^2(1, 1) = \frac{\sqrt{3}}{2}(\boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2)_\mu^2 \quad (18)$$

$$S_\mu^1(0, 1) = \frac{1}{2\sqrt{2}}(\boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2)_\mu^1 - \frac{1}{4}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)_\mu^1 \quad (19)$$

$$S_\mu^1(1, 0) = \frac{\sqrt{3}}{2\sqrt{2}}(\boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2)_\mu^1 + \frac{\sqrt{3}}{4}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)_\mu^1. \quad (20)$$

Thus, the double differential cross section is given by

$$\frac{d^2\sigma_0}{dW d\Omega_f d\Omega} = Tr[\rho^f] = P_0^0(0, 0) + P_0^0(1, 1), \quad (21)$$

in terms of the double differential cross sections $P_0^0(0, 0)$ leading to the final singlet state and $P_0^0(1, 1)$ leading to the final triplet state of the two protons. If we use the notations $(\mathbf{P}_i)_\mu$ to denote the spherical components, i.e.,

$$(\mathbf{P}_i)_0 = P_{iz}; (\mathbf{P}_i)_{\pm 1} = \mp \frac{1}{\sqrt{2}}(P_{ix} \pm P_{iy}), \quad (22)$$

it follows from (19) and (20) that

$$P_\mu^1(1, 0) - \sqrt{3}P_\mu^1(0, 1) = \frac{\sqrt{3}}{2}Tr[\rho^f](\mathbf{P}_1 - \mathbf{P}_2)_\mu, \quad (23)$$

where as it follows from (17) that

$$P_\mu^1(1, 1) = \frac{\sqrt{3}}{2\sqrt{2}}Tr[\rho^f](\mathbf{P}_1 + \mathbf{P}_2)_\mu, \quad (24)$$

which together determine \mathbf{P}_1 and \mathbf{P}_2 individually. Finally, the spin correlations $C_{\alpha\beta}$ defined in (13) may like wise be related to (14) using

$$P_0^0(1, 1) - 3P_0^0(0, 0) = Tr[(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \rho^f], \quad (25)$$

$$P_\mu^1(1, 0) + \sqrt{3}P_\mu^1(0, 1) = \frac{\sqrt{3}i}{2}Tr[\rho^f](\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)_\mu \quad (26)$$

$$P_\mu^2(1, 1) = \frac{\sqrt{3}}{2}Tr[\rho^f](\boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2)_\mu^2. \quad (27)$$

Using the known properties [18] of the spin operators S_μ^λ and standard Racah techniques, we may obtain a master formula for all the final state spin observables, which is given by

$$P_\mu^k(s'_f, s_f) = \frac{1}{4} \sum_{s_i, \lambda, \lambda'} (-1)^{s_f - s_i} [s_f] [s'_f]^2 [\lambda] [\lambda'] \times W(s'_f \lambda' s_f \lambda; s_i k) \times (M^\lambda(s_f, s_i) \otimes M^{\dagger \lambda'}(s'_f, s_i))_\mu^k, \quad (28)$$

where $M_\mu^{\dagger \lambda}(s_f, s_i)$ are defined in terms of the complex conjugates $M_\mu^\lambda(s_f, s_i)^*$ of $M_\mu^\lambda(s_f, s_i)$ given by (2) through

$$M_\mu^{\dagger \lambda}(s_f, s_i) = (-1)^\mu M_{-\mu}^\lambda(s_f, s_i)^*, \quad (29)$$

Noting once again that $(-1)^{l_f + l + l_i} = -1$, due to parity conservation, we may express

$$M_\mu^{\dagger \lambda}(s_f, s_i) = (-1)^{1-\lambda} \sum_{\mathcal{L}} W(l_i s_i L_f s_f; j \lambda) \times Z^*(s_f, s_i, j, \mathcal{L}) A_\mu^\lambda(\mathcal{L}), \quad (30)$$

where $Z^*(s_f, s_i, j, \mathcal{L})$ denote the complex conjugates of $Z(s_f, s_i, j, \mathcal{L})$ given by (4).

IV. RELATIVE PHASE OF THE THRESHOLD AMPLITUDE

We may now take advantage of the fact that $M_0^1(0, 1)$ given by (10) is spherically symmetric with respect to $\hat{\mathbf{p}}_f$ and $\hat{\mathbf{q}}$ and involves only the threshold amplitude f_1 . Moreover, $M_\mu^\lambda(1, 1)$ are independent of f_1 and depend only on the Pp amplitudes f_4, \dots, f_{12} . Therefore, we focus attention on (26) and (23) which involve

$$(M^\lambda(1, 1) \otimes M^{\dagger 1}(0, 1))_\mu^1 = \frac{1}{4\sqrt{3}\pi} \sum_{\mathcal{L}, j} W(l_i 1 L_f 1; j \lambda) \times Z(1, 1, j, \mathcal{L}) f_1^* \times (A^\lambda(\mathcal{L}) \otimes Y_1(\hat{\mathbf{p}}_i))_\mu^1, \quad (31)$$

$$(M^1(0, 1) \otimes M^{\dagger \lambda}(1, 1))_\mu^1 = \frac{-1}{4\sqrt{3}\pi} \sum_{\mathcal{L}, j} W(l_i 1 L_f 1; j \lambda) \times Z^*(1, 1, j, \mathcal{L}) f_1 \times (A^\lambda(\mathcal{L}) \otimes Y_1(\hat{\mathbf{p}}_i))_\mu^1, \quad (32)$$

Expressing

$$(A^\lambda(\mathcal{L}) \otimes Y_1(\hat{\mathbf{p}}_i))_\mu^1 = \frac{\sqrt{3}}{4\pi} \sum_{L_i} W(L_f l_i 1 1; \lambda L_i) \times [\lambda] [L_i] [l_i] C(l_i 1 L_f 000) \times A_\mu^1(l_f l L_f L_i), \quad (33)$$

and carrying out the summation over \mathcal{L} and j , we obtain

$$P_\mu^1(1,0) = f_1^*[F_1 A_\mu^1(1110) + F_2 A_\mu^1(1112) + F_3 A_\mu^1(1122)], \quad (34)$$

$$P_\mu^1(0,1) = \frac{-1}{\sqrt{3}} f_1[F_1^* A_\mu^1(1110) + F_2^* A_\mu^1(1112) + F_3^* A_\mu^1(1122)], \quad (35)$$

where $F_i, i = 1, 2, 3$ are well-defined linear combinations of the Pp amplitudes given by

$$F_1 = \frac{1}{32\pi^{3/2}} [f_4 - \frac{5}{6}f_5 + \frac{5}{2\sqrt{3}}f_6 + \frac{1}{3\sqrt{3}}f_9 - \frac{1}{6}f_{10} - \frac{\sqrt{5}}{6\sqrt{3}}f_{11}], \quad (36)$$

$$F_2 = \frac{1}{32\sqrt{2}\pi^{3/2}} [f_5 - \sqrt{3}f_6 + \sqrt{\frac{3}{2}}f_7 + \frac{3}{\sqrt{2}}f_8], \quad (37)$$

$$F_3 = \frac{-1}{32\sqrt{2}\pi^{3/2}} [\sqrt{3}f_5 + f_6 + \sqrt{7}f_7 + \sqrt{\frac{7}{3}}f_8], \quad (38)$$

Since the Pp amplitudes have been determined both in magnitude and relative phase w.r.t f_2 in [20], we may express

$$F_\alpha = |F_\alpha| e^{i\Delta_\alpha}, \alpha = 1, 2, 3 \quad (39)$$

and treat $|F_\alpha|$ and Δ_α as known. In [20], f_2 was assumed to be real. Since the relative phase between f_1 and f_2

could not be ascertained from the measurements of Meyer et al. [15], f_1 was also assumed to be real, although only one of the amplitudes can be taken as real. Therefore, we choose f_2 to be real and express f_1 as

$$f_1 = |f_1| e^{i\delta_1}, \quad (40)$$

This leads to

$$P_\mu^1(1,0) - \sqrt{3}P_\mu^1(0,1) = 2 \sum_{\alpha=1}^3 R_\alpha \cos(\Delta_\alpha - \delta_1) A_\mu^1(\alpha), \quad (41)$$

$$P_\mu^1(1,0) + \sqrt{3}P_\mu^1(0,1) = 2i \sum_{\alpha=1}^3 R_\alpha \sin(\Delta_\alpha - \delta_1) A_\mu^1(\alpha), \quad (42)$$

where $R_\alpha = |F_\alpha| |f_1|$ and $A_\mu^1(\alpha)$ for $\alpha = 1, 2, 3$ denote $A_\mu^1(1110), A_\mu^1(1112), A_\mu^1(1122)$ respectively.

It is seen from (11) that measuring the double differential crosssection (7) yields $Tr[\rho^f]$. Measurements of $(\mathbf{P}_1 - \mathbf{P}_2)_\mu$ given by (23) then leads to empirical determination of (41), while measurements of spin correlations $C_{xy} - C_{yx}, C_{yz} - C_{zy}, C_{zx} - C_{xz}$ where $C_{\alpha\beta}$ are given by (13) lead to empirical determination of (42) using (26).

Thus, we find that it is possible to determine empirically the relative phase δ_1 of f_1 , without any trigonometric ambiguities, since R_α and Δ_α are known from [20]. We therefore advocate measurement of these pp spin observables in the final state, employing simply an unpolarized beam and unpolarized target initially, to complement the spin observables measured by Meyer et al.[15], so that the amplitudes f_1, f_2, \dots, f_{12} may be determined empirically without any phase ambiguity.

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